MATHEMATICS STANDARDS
BY LEVEL

PROFICIENCY (Grades 9-12)

STANDARD 1: NUMBER SENSE

Students develop number sense and use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to determine the reasonableness of results.

- **1M-P1. Compare and contrast the real number system and its various subsystems with regard to their structural characteristics**

Core – will be tested on AIMS

PO 1. Classify numbers as members of the sets (natural, whole, integers, rationals and irrationals)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 2. Compare subsets of the real number system with regard to their properties (commutative, associative, distributive, identity, inverse and closure properties)
PO 4. Identify whether a given set of numbers is finite or infinite
{PO 3 Deleted}

- **1M-P2. Construct, interpret and demonstrate meaning for real numbers and absolute value in problem-solving situations**

Core – will be tested on AIMS

PO 1. Determine a rational estimate of an irrational number
PO 3. Solve real-world distance problems using absolute value
PO 4. Determine, among the solutions to a real-world problem, which, if any, is reasonable
PO 6. Choose the appropriate signed real number to represent a real-world value
PO 7. Use the appropriate form of a real number to express a real-world situation (e.g., choosing between a radical expression or rational approximation)
PO 8. Convert standard notation to scientific notation, including negative exponents, and vice versa

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 2. Define *absolute value* as the distance from the origin
{PO 5 Moved to 4M-P4, PO 4}

STANDARD 2: DATA ANALYSIS AND PROBABILITY

Students use data collection and analysis, statistics, and probability to make valid inferences, decisions and arguments and to solve a variety of real-world problems.

- **2M-P1. Construct and draw inferences including measures of central tendency, from charts, tables, graphs and data plots that summarize data from real-world situations**

Core – will be tested on AIMS

PO 1. Organize collections of data into frequency charts, stem-and-leaf plots, scatter plots
PO 2. Construct histograms, line graphs, circle graphs and box-and-whisker plots
PO 4. Evaluate the reasonableness of conclusions drawn from data analysis
PO 5. Use mean, median, mode, quartiles and range as a means for effective decision making in analyzing the data and the outliers
PO 6. Identify graphic misrepresentations and distortions of sets of data (e.g., omissions of parts of axis range, unequal interval sizes)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 3. Draw inferences from collections of data

- **2M-P2. Use appropriate technology (e.g., graphing calculators, computer software) to display and analyze data**

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Use appropriate technology to display data as lists, tables, matrices and plots
PO 2. Use appropriate technology to calculate mean, median, mode, minimum and maximum
PO 3. Use appropriate technology to predict patterns in sets of data (e.g., "Does a scatter plot appear to be linear?")

- **2M-P3. Apply curve fitting to make predictions from data**

Core – will be tested on AIMS

PO 1. Draw a line which closely fits a scatter plot
PO 2. Make a prediction from a linear pattern in plots of data

*Beyond Core**

PO 1. Draw a curve which closely fits a scatter plot

*Beyond Core: Appropriate to be taught after a grounding in core instruction, but will not be tested on AIMS

- **2M-P4. Explain the effects of sampling on statistical claims and recognize misuses of statistics**

Core – will be tested on AIMS

PO 1. Differentiate between sampling and census
PO 2. Differentiate between a biased and an unbiased sample
PO 3. Recognize the impact of interpreting data from a biased sample

Beyond Core

PO 4. Distinguish the effects of using statistical measures obtained from a sample vs. those obtained from a census
PO 5. Recognize the misinterpretations of data from different representations of those same data
PO 6. Determine the validity of sampling methods in studies

- **2M-P5. Design and conduct a statistical experiment to study a problem and interpret and communicate the outcomes**

Beyond Core

PO 1. Design a statistical experiment based on a given hypothesis
PO 2. Create an appropriate data-gathering instrument (e.g., biased vs. unbiased questions, multiple choice vs. open-ended)
PO 3. Organize collected data into an appropriate graphical representation
PO 4. Draw and support inferences that are based on data analysis

- **2M-P6. Use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty**

Beyond Core

PO 1. Recognize whether experimental or theoretical methods were used to calculate a particular probability
PO 2. Use experimental observations to estimate probabilities of entire populations
PO 3. Distinguish between independent and dependent events
PO 4. Solve probability problems involving *and* and *or* statements, with and without replacement

- **2M-P7. Use simulations to estimate probabilities**

Beyond Core

PO 1. Design appropriate simulations to estimate probabilities of real-world situations (e.g., disk toss, cube toss, technological simulations)
PO 2. Use simulations to estimate probabilities of real-world situations

- **2M-P8. Solve real-world problems by using combinations and permutations**

Core – will be tested on AIMS

PO 1. Use a tree diagram or a chart of possible outcomes to count probable outcomes of an event

Beyond Core

PO 2. Determine when to use combinations in counting objects
PO 3. Determine when to use permutations in counting objects
PO 4. Use combinations and permutations to solve real-world problems not requiring the use of formulas

- **2M-P9. Describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed**

Beyond Core

PO 1. Determine if data gathered from a real-world situation fits a normal curve
PO 2. Describe the central tendency characteristics of the normal curve
PO 3. Make simple predictions from data represented on a given normal curve

- **2M-P10. Explain the concept of a random variable**

Beyond Core

PO 1. Distinguish situations where a random variable is needed or used
{PO 2 Deleted}

- **2M-P11. Apply measures of central tendency, variability and correlation**

Core – will be tested on AIMS

PO 1. Apply the concepts of mean, median, mode and range to draw conclusions about data
PO 3. Determine, from a given plot of data, whether it has positive or negative correlation

Beyond Core

PO 2. Draw conclusions about the "spread" of data given the variance and standard deviation (e.g., compare sets of data with the same central tendency, but with different variance)

STANDARD 3: PATTERNS, ALGEBRA AND FUNCTIONS

Students use algebraic methods to explore, model and describe patterns, relationships and functions involving numbers, shapes, data and graphs within a variety of real-world problem solving situations.

- **3M-P1. Model real-world phenomena (e.g., compound interest or the flight of a ball) using functions and relations (e.g., linear, quadratic, sine and cosine, and exponential)**

  Core – will be tested on AIMS

  PO 2. Describe a real-world situation that is depicted by a given graph

  Beyond Core*

  PO 1. Identify the independent and dependent variables from a real-world situation

  Core – to be taught in grades 9-10, but will not be tested on AIMS

  PO 3. Sketch a graph that models a given real-world situation

- **3M-P2. Represent and analyze relationships using written and verbal explanations, tables, equations, graphs and matrices and describe the connections among those representations**

  Core – will be tested on AIMS

  PO 3. Determine whether a relation is a function, given the graphical representation

  Core – to be taught in grades 9-10, but will not be tested on AIMS

  PO 1. Express the relationship between two variables using a table, equation, graph and matrix

  {PO 2 Deleted}

- **3M-P3. Analyze the effects of parameter changes on functions (e.g., linear, quadratic and trigonometric) using calculators and/or computers**

  Beyond Core

  PO 1. Use technology to determine changes in the shape and behavior of polynomial functions (of degree 2 or less) when constants and coefficients are varied

*Beyond Core: Appropriate to be taught after a grounding in core instruction, but will not be tested on AIMS

- **3M-P4. Interpret algebraic equations and inequalities geometrically and describe geometric relationships algebraically**

Core – will be tested on AIMS

PO 1. Graph a linear equation in two variables
PO 2. Graph a linear inequality in two variables
PO 3. Determine slope and intercepts of a linear equation
PO 4. Write an equation of the line that passes through two given points
PO 5. Determine from two linear equations whether the lines are parallel, are perpendicular or coincide

- **3M-P5. Apply trigonometry to real-life problem situations (e.g., investigate how to find the distance across a river using similar triangles and trigonometric ratios; compare the sine and cosine curves to the curves of sound waves)**

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Use the definitions of trigonometric functions to find the sine, cosine and tangent of the acute angles of a right triangle

Beyond Core

PO 2. Solve simple right-triangle trigonometric equations involving sine, cosine and tangent
PO 3. Use an appropriate right-triangle trigonometric model to solve a real-life problem

- **3M-P6. Perform mathematical operations on expressions and matrices, and solve equations and inequalities**

Core – will be tested on AIMS

PO 1. Simplify numerical expressions using the order of operations, including exponents
PO 2. Evaluate algebraic expressions using substitution
PO 3. Simplify algebraic expressions using distributive property
PO 4. Simplify square roots and cube roots with monomial radicands that are perfect squares or perfect cubes
PO 6. Evaluate numerical and algebraic absolute value expressions
PO 7. Multiply and divide monomial expressions with integer exponents
PO 9. Solve linear equations and inequalities in one variable
PO 10. Solve formulas for specified variables
PO 11. Solve quadratic equations (integral roots only)
PO 13. Solve proportions which generate linear equations
PO 15. Solve systems of linear equations in two variables (integral coefficients and solutions)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 5. Calculate powers and roots of real numbers, both rational and irrational, using technology

PO 14. Solve absolute value equations containing a single absolute value expression

Beyond Core

PO 8. Add, subtract and perform scalar multiplication with matrices

PO 12. Solve radical equations involving one radical (restrict to square roots)

- **3M-P7. Translate among tabular, symbolic and graphical representations of functions**

Core – will be tested on AIMS

PO 1. Create a linear equation from a table of values

PO 2. Create a graph from a table of values

PO 3. Determine the solution to a system of equations in two variables, from a given graph

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 4. Determine the solution to a system of inequalities in two variables, from a given graph (e.g., "Which of the shaded regions represents the solution to the system?")

- **3M-P8. Use the power of mathematical abstraction and algebraic symbolism to represent various situations**

Core – will be tested on AIMS

PO 1. Translate verbal expressions and sentences to mathematical expressions and sentences

PO 2. Generate an algebraic sentence to model real-life situations, given a data set (limited to linear relationships)

- **3M-P9. Determine maximum and minimum points of a graph and interpret results in problem situations**

Core – will be tested on AIMS

PO 2. Determine domain and range of a relation, given the graph or a set of points

{PO 1 and PO 3 Deleted}

- **3M-P10. Investigate the limiting process by examining infinite sequences and series and areas under curves**

Beyond Core

PO 1. Compare the estimates of the area under a curve over a bounded interval, using progressively smaller rectangles (not using calculus)
PO 2. Estimate the limit of a given infinite sequence (e.g., given the sequence 1/*n*, as *n* gets larger) (not using calculus)

STANDARD 4: GEOMETRY

Students use geometric methods, properties and relationships as a means to recognize, draw, describe, connect and analyze shapes and representations in the physical world.

- **4M-P1. Interpret and draw three-dimensional objects**

Core – will be tested on AIMS

PO 1. Sketch prisms, pyramids, cones, cylinders and spheres
PO 2. Classify prisms, pyramids, cones, cylinders and spheres by base shape and lateral surface shape
PO 3. Recognize the three-dimensional figure represented by a two-dimensional drawing (e.g., "What figures are represented by given nets, sketches, photographs?")

- **4M-P2. Represent problem situations with geometric models and apply properties of figures**

Core – will be tested on AIMS

PO 1. Calculate surface areas and volumes of three-dimensional geometric figures, given the required formulas
PO 2. Solve applied problems using angle and side length relationships
PO 3. Solve applied problems using the Pythagorean theorem (e.g., determine whether a wall is square)
PO 4. Solve applied problems using congruence and similarity relationships of triangles (e.g., estimate the height of a building, using shadows)
PO 6. Determine the distance and midpoint between points within a coordinate system representative of a practical application
PO 7. Find the area of a geometric figure composed of a combination of two or more geometric figures, given an appropriate real-world situation and the formulas
PO 8. Solve problems involving complementary, supplementary and congruent angles

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 5. Make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object (models and representations include scale drawings, perspective drawings, blueprints or computer simulations)

- **4M-P3. Deduce properties of figures using transformations in coordinate systems, identifying congruency and similarity**

Core – will be tested on AIMS

PO 1. Determine whether a planar figure is symmetric with respect to a line
PO 3. Determine the effects of a transformation on linear and area measurements of the original planar figure
PO 4. Sketch the planar figure that is the result of a given transformation

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 2. Give the new coordinates of a transformed geometric planar figure

- **4M-P4. Deduce properties of, and relationships between, figures from given assumptions**

Core – will be tested on AIMS

PO 1. Find similarities and differences among geometric shapes and designs using a given attribute (e.g., height, area, perimeter, diagonals and angle measurements)
PO 2. Identify arcs, chords, tangents and secants of a circle
PO 3. State valid conclusions using given geometric definitions, postulates and theorems
PO 4. Represent *pi* as the ratio of circumference to diameter {Moved from 1M-P2, PO5}

- **4M-P5. Translate between synthetic and coordinate representations (e.g., a straight line is represented by the algebraic equation $Ax + By = C$)**

Core – will be tested on AIMS

PO 1. Determine the relative placement of two lines on a coordinate plane by examining the algebraic equations representing them

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 2. Verify characteristics of a given geometric figure using coordinate formulas such as distance, mid-point, and slope to confirm parallelism, perpendicularity and congruency

- **4M-P6. Recognize and analyze Euclidean transformations (e.g., reflections, rotations, dilations and translations)**

Core – will be tested on AIMS

PO 1. Classify transformations based on whether they produce congruent or similar non-congruent figures

PO 2. Determine whether a given pair of figures on a coordinate plane represents a translation, reflection, rotation and/or dilation

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 3. Apply transformational principles to practical situations (e.g., enlarge a photograph)

STANDARD 5: MEASUREMENT AND DISCRETE MATHEMATICS

Students make and use direct and indirect measurement, metric and U.S. customary, to describe and compare the real world and to prepare for the study of discrete functions, fractals and chaos which have evolved out of the age of technology.

- **5M-P1. Represent problem situations using discrete structures such as finite graphs, matrices, sequences and recurrence relations**

*Beyond Core**

PO 1. Use matrices and finite graphs to display data

PO 2. Find a specified n^{th} term of a simple arithmetic or geometric sequence, where the common difference or common ratio is an integer and $n > 100$

PO 3. Use simple or basic recursion formulas to solve real-life problems (e.g., compound interest)

- **5M-P2. Represent and analyze finite graphs using matrices**

Beyond Core

PO 1. Interpret data using matrices and finite graphs (e.g., networks, street diagrams, tournament schedules, production schedules)

PO 2. Determine when a finite graph gives an accurate picture of a data set

PO 3. Translate a finite graph into a matrix and vice versa

*Beyond Core: Appropriate to be taught after a grounding in core instruction, but will not be tested on AIMS

- **5M-P3. Develop and analyze algorithms**

Core – will be tested on AIMS

PO 2. Determine the purpose of a given algorithm (simple, basic **math** algorithm)
PO 3. Determine whether given algorithms are equivalent (simple, basic **math** algorithms)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Write an algorithm that explains a particular mathematical process (e.g., tell a younger child how to find the average of two numbers)

- **5M-P4. Solve enumeration and finite probability problems**

Core – will be tested on AIMS

PO 1. Find the outcome set of a situation
PO 2. Find the probability that a specific event will happen
PO 4. Determine the number of possible outcomes in a real-world situation using the counting principle and tree diagrams

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 3. Determine theoretical geometrical probabilities, given necessary formulas (e.g., "Given a circular target on a square base, what is the probability of hitting the circle with a dart, providing the dart goes inside the square?")
{PO 5 Deleted}

STANDARD 6: MATHEMATICAL STRUCTURE/LOGIC

Students use both inductive and deductive reasoning as they make conjectures and test the validity of arguments.

- **6M-P1. Use inductive and deductive logic to construct simple valid arguments**

Core – will be tested on AIMS

PO 2. Produce a valid conjecture using inductive reasoning by generalizing from a pattern of observations (e.g., if $10^1 = 10$, $10^2 = 100$, $10^3 = 1000$, make a conjecture)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Construct a simple informal deductive proof (e.g., write a proof of the statement: "Given an airline schedule with cities and flight times, you can fly from Bombay to Mexico City")

- **6M-P2. Determine the validity of arguments**

Core – will be tested on AIMS

PO 2. Draw a simple valid conclusion from a given *if . . . then* statement and a minor premise
PO 3. Distinguish valid arguments from invalid arguments
PO 4. List related *if . . . then* statements in logical order

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Determine if the converse of a given statement is true or false
PO 6. Analyze assertions about everyday life by using principles of logic (e.g., examine the fallacies of advertising)

*Beyond Core**

PO 7. Recognize the difference between a statement verified by mathematical proof (i.e., a theorem) and one verified by empirical data (e.g., women score higher than men on vocabulary tests)
{PO 5 Deleted}

- **6M-P3. Formulate counterexamples and use indirect proof**

Core – will be tested on AIMS

PO 1. Construct a counterexample to show that a given invalid conjecture is false (e.g., Nina makes a conjecture that $x^3 > x^2$ for all values of x. Find a counterexample.)

- **6M-P4. Make and test conjectures**

Beyond Core

PO 1. Write an appropriate conjecture given a certain set of circumstances
PO 2. Test a conjecture by constructing a logical argument or a counterexample

- **6M-P5. Understand the logic of algebraic procedures**

Core – will be tested on AIMS

PO 1. Determine whether a given algebraic expression and a possible simplified form are equivalent (e.g., show that $(x + y)^2 = x^2 + y^2$ is invalid)
PO 2. Determine whether a given procedure for solving an equation is valid

*Beyond Core: Appropriate to be taught after a grounding in core instruction, but will not be tested on AIMS